

Exercise 17

For the following exercises, use synthetic division to find the quotient.

$$(4x^3 - 12x^2 - 5x - 1) \div (2x + 1)$$

Solution

Synthetic division only works if the divisor is of the form $x - k$. Multiply the numerator and denominator by $1/2$ to make it so.

$$\frac{4x^3 - 12x^2 - 5x - 1}{2x + 1} = \frac{4x^3 - 12x^2 - 5x - 1}{2x + 1} \times \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{2x^3 - 6x^2 - \frac{5}{2}x - \frac{1}{2}}{x + \frac{1}{2}}$$

Solving $x + \frac{1}{2} = 0$ gives $x = -\frac{1}{2}$; this is the number that goes on the top left. Write out all the coefficients of the dividend to the right.

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -6 & -\frac{5}{2} & -\frac{1}{2} \\ \hline & & & & \end{array}$$

Bring down the leading coefficient.

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -6 & -\frac{5}{2} & -\frac{1}{2} \\ \hline & 2 & & & \end{array}$$

Multiply the top left number by the number brought down and put the result under the second coefficient of the dividend.

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & -6 & -\frac{5}{2} & -\frac{1}{2} \\ \hline & 2 & -1 & & \end{array}$$

Add the numbers in the second column.

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & -6 & -\frac{5}{2} & -\frac{1}{2} \\ \hline & & & & \\ & & & -1 & \\ \hline & 2 & -7 & & \end{array}$$

Multiply this sum of the second column by the top left number and put it in the next column.

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & -6 & -\frac{5}{2} & -\frac{1}{2} \\ \hline & & & & \\ & & & 7 & \\ & & -1 & \frac{7}{2} & \\ \hline & 2 & -7 & & \end{array}$$

Add the numbers in the third column.

$$\begin{array}{c|cccc} \frac{1}{2} & 2 & -6 & -\frac{5}{2} & -\frac{1}{2} \\ \hline & & & & \\ & & & 7 & \\ & & -1 & \frac{7}{2} & \\ \hline & 2 & -7 & 1 & \end{array}$$

Multiply this sum of the third column by the top left number and put it in the next column.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -6 & -\frac{5}{2} & -\frac{1}{2} \\ \hline & & -1 & \frac{7}{2} & -\frac{1}{2} \\ \hline & 2 & -7 & 1 & \end{array}$$

Add the numbers in the fourth column.

$$\begin{array}{r|rrrr|r} \frac{1}{2} & 2 & -6 & -\frac{5}{2} & -\frac{1}{2} \\ \hline & & -1 & \frac{7}{2} & -\frac{1}{2} \\ \hline & 2 & -7 & 1 & -1 \end{array}$$

This final result is the remainder, and the numbers to the left are the coefficients of the quotient, which is $2x^2 - 7x + 1$.

$$(4x^3 - 12x^2 - 5x - 1) \div (2x + 1) = 2x^2 - 7x + 1 + \frac{-1}{x + \frac{1}{2}}$$